

Generalized Energy Based Models

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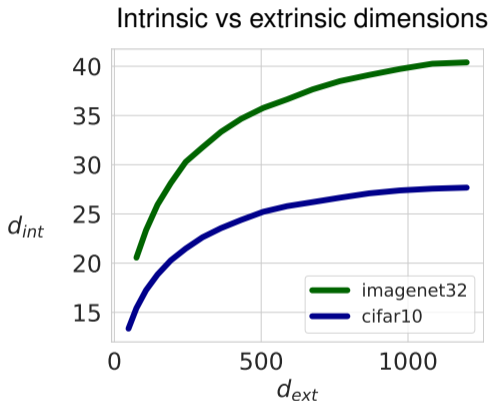
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March 25, 2021

Motivation

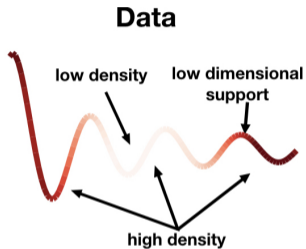
Problem

- ✓ Setting: data distributions with *small intrinsic dimension* embedded in a space with *high extrinsic dimension*.
- ✓ Example: Includes data such as natural images (Thiry et al. 2021).
- ✓ Goal: Flexible models exploiting low intrinsic dimensionality.



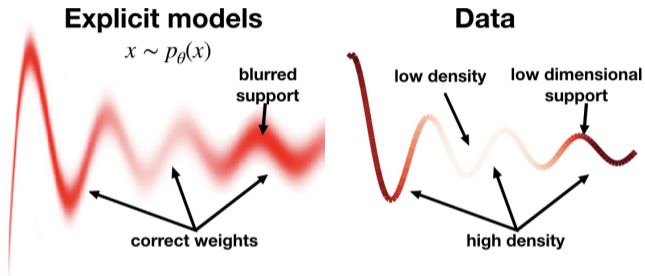
Motivation

Modeling data with **low dimensional support** and **multiple modes**.



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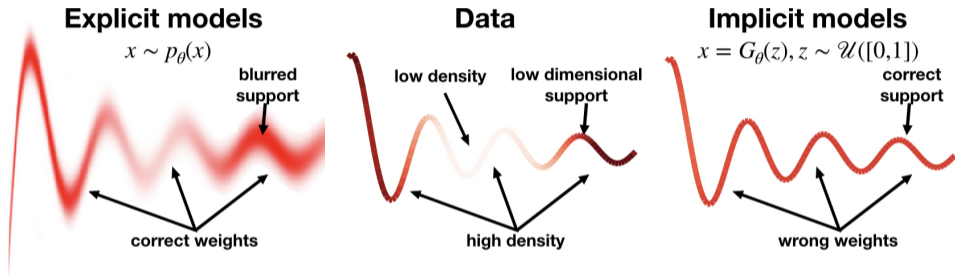
Modeling data with **low dimensional support** and **multiple modes**.



- ▶ An *explicit model* puts mass on the whole space: it blurs the samples.

Motivation

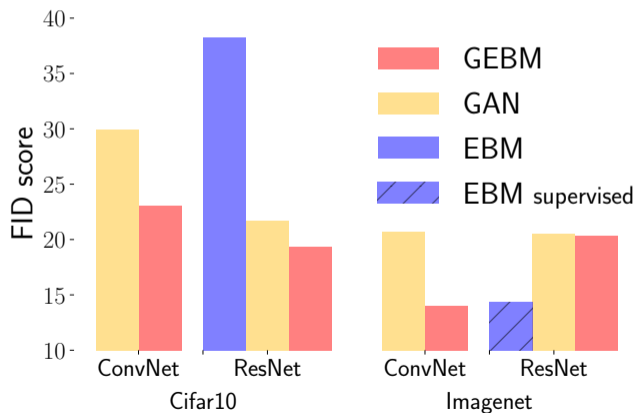
Modeling data with **low dimensional support** and **multiple modes**.



- ▶ *Explicit models* put mass on the whole space: blurring effect.
- ▶ *Implicit models* are **wasteful**: it throws the critic away (Azadi et al. 2019).

GEBMs: A new class of models for data with low intrinsic dimension

- ✓ Combines Implicit and Explicit models.
- ✓ Improves over GANs by using the critic information for sampling.
- ✓ Improves over EBMs by allowing their support to be **learnable** and **low-dimensional**.



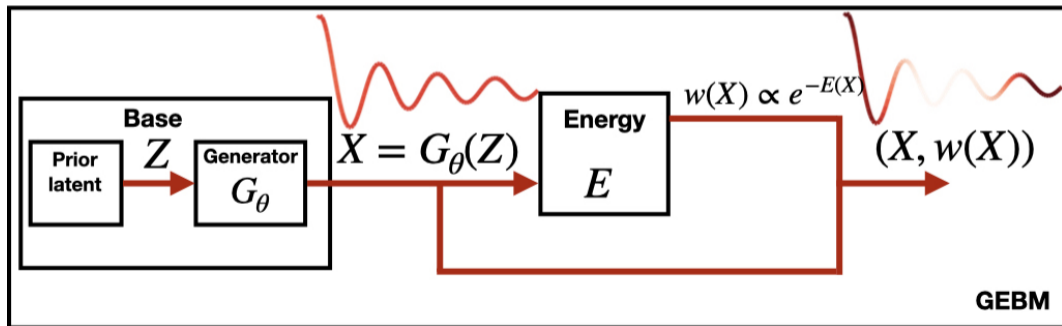
Generalized energy-based models

GEBMs are defined by a combination of the two components: *energy* and *base*

- ▶ The **base** \mathbb{G}_θ maps a **prior latent** noise $Z \sim \eta$ using a **generator** G_θ
- ▶ The **energy** E defines importance weights on the support of \mathbb{G}_θ

$$X \sim \mathbb{G}_\theta \iff X = G_\theta(Z), \quad Z \sim \eta$$

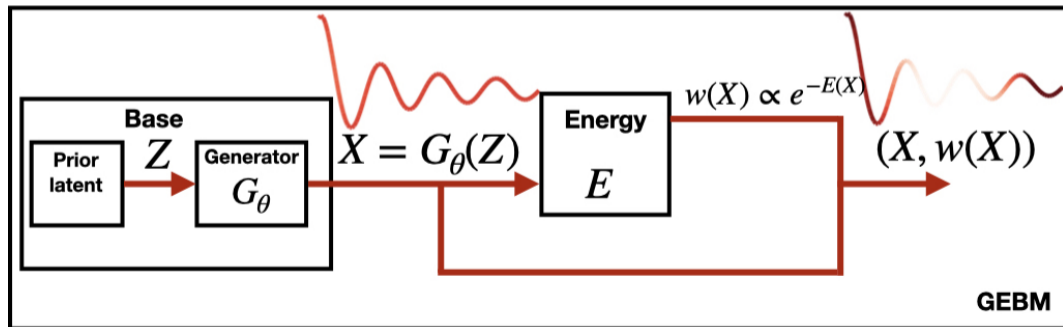
$$w(X) \propto \exp(-E(X))$$



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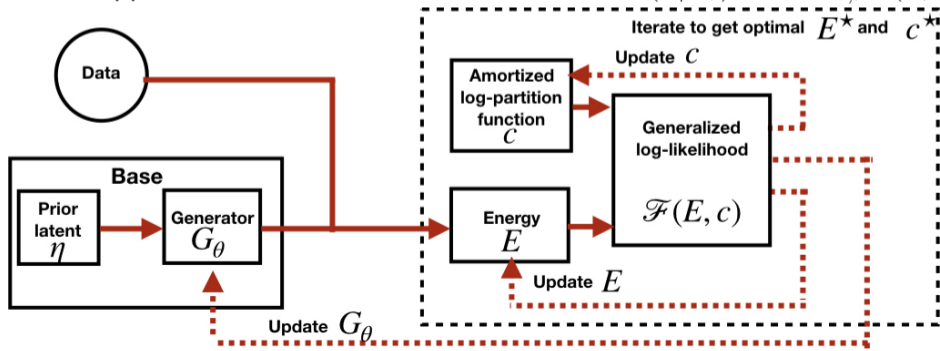
- ▶ The **base** learns the low-dimensional support of the data.
- ▶ The **energy** refines the mass on the low-dimensional support of the base.



Adversarial Training for GEBMs

Training with computational cost as GANs Goodfellow et al. 2014 and alternates between two stages

- ▶ **Energy** E : analogous to a critic in an f -GAN (Nowozin et al. 2016) and is learned by maximum likelihood with **amortized estimation** of the log-partition c of the model: $\max_{E,c} \mathcal{F}(E, c)$.
- ▶ **Base** \mathbb{G}_θ : analogous to a generator in an f -GAN and is learned by minimizing the **KL Approximate Lower-bound Estimator** $KALE(\mathbb{P}|\mathbb{G}_\theta) := \max_{E,c} \mathcal{F}(E, c)$.



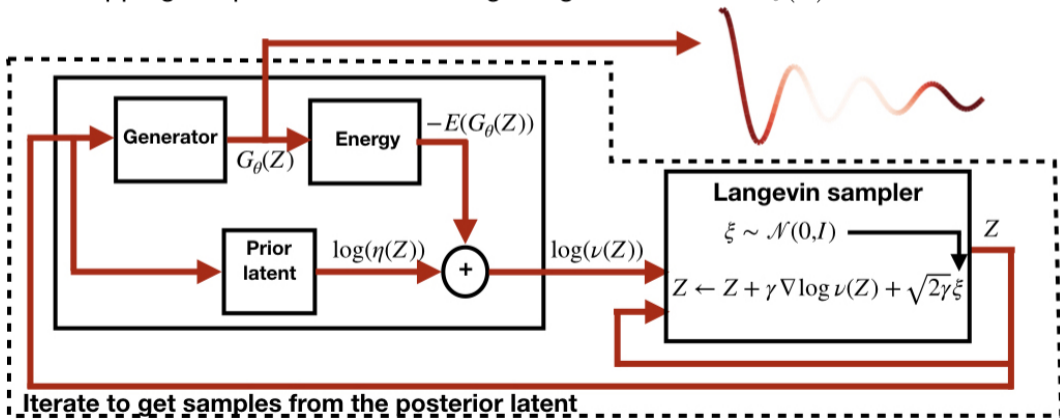
Sampling from GEBMs using Latent MCMC

Sampling from GEBM requires:

- ▶ Producing a **posterior latent** noise $Z \sim \nu$ using MCMC with

$$\nu(Z) \propto \eta(Z)e^{-E(G_\theta(Z))}$$

- ▶ Mapping the posterior noise using the generator $X = G_\theta(Z)$.

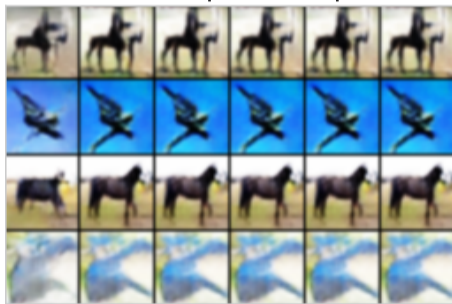


Sampling from GEBMs using Latent MCMC

Various MCMC sampler can be used and yield different behaviors.

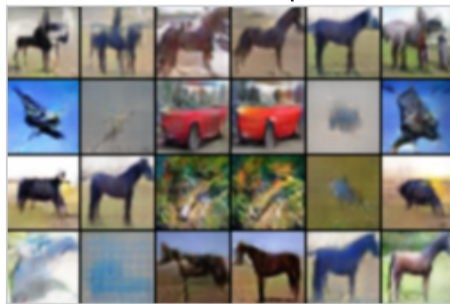
- ▶ Overdamped samplers (like ULA) stick to a particular mode within each chain.
- ▶ Kinetic samplers (like HMC) explore multiple modes within the same chain.

Overdamped sampler



t=0 t=20 t=40 t=60 t=80 t=100

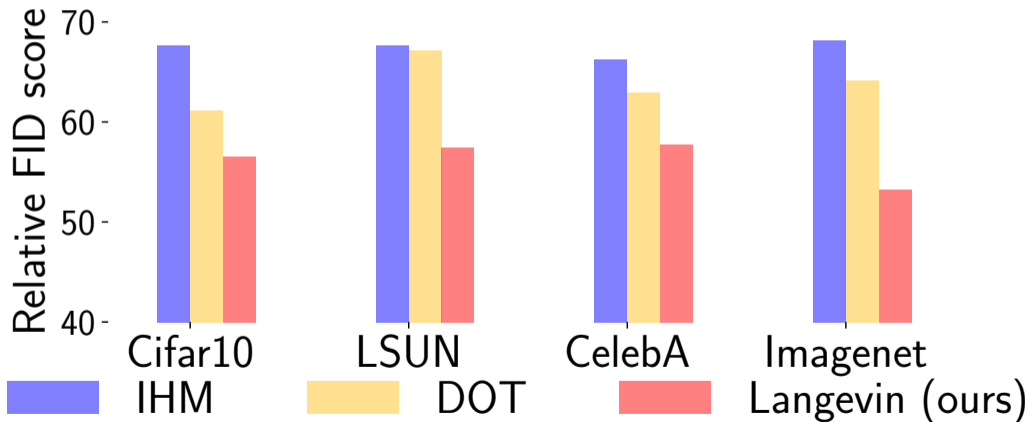
Kinetic sampler



t=0 t=20 t=40 t=60 t=80 t=100

Sampling from GEBMs using Latent MCMC

- ✓ Sampling using MCMC in latent space exploits gradient information of the discriminator to accelerate mixing.



Conclusion

GEBM: A new family of models bridging the gap between GANs and EBMs.

- ✓ Modeling data with small intrinsic dimension:
 - ▶ The base learns the low dimensional support of data
 - ▶ The energy refines the distribution of mass on the base.
- ✓ Adversarial training
- ✓ Latent MCMC sampling
- ✓ Improves over GANs and EBMs

Code:

<https://github.com/MichaelArbel/GeneralizedEBM>

Thank you !



Azadi, Samaneh et al. (2019). “Discriminator Rejection Sampling”. In: [arXiv:1810.06758 \[cs, stat\]](https://arxiv.org/abs/1810.06758). arXiv: 1810.06758. URL: <http://arxiv.org/abs/1810.06758> (visited on 02/06/2020).



Goodfellow, Ian J. et al. (2014). “Generative Adversarial Networks”. In: [arXiv:1406.2661 \[cs, stat\]](https://arxiv.org/abs/1406.2661). arXiv: 1406.2661. URL: <http://arxiv.org/abs/1406.2661> (visited on 03/18/2019).



Nowozin, Sebastian, Botond Cseke, and Ryota Tomioka (2016). “f-GAN: Training Generative Neural Samplers using Variational Divergence Minimization”. In: [eprint: 1606.00709](https://arxiv.org/abs/1606.00709). URL: <https://arxiv.org/abs/1606.00709>.



Thiry, Louis et al. (2021). “The Unreasonable Effectiveness of Patches in Deep Convolutional Kernels Methods”. In: [ICLR](#).