

# Efficient Wasserstein Natural Gradients for Reinforcement Learning

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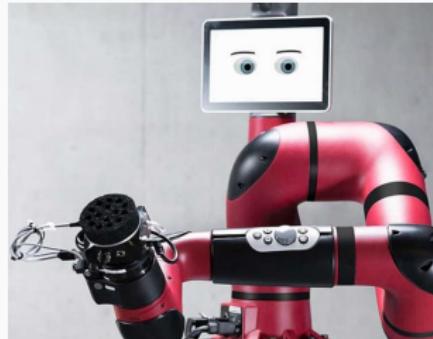
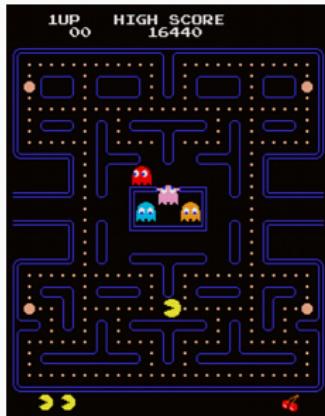


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# Motivation

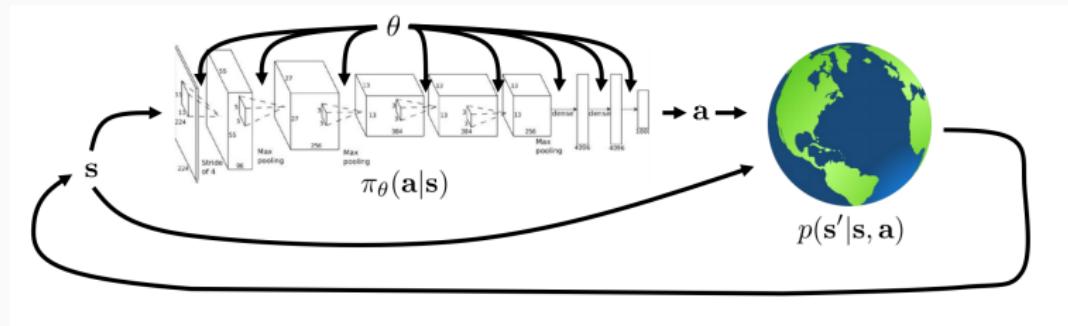


- Deep RL algorithms require  $\sim 10^6\text{-}10^9$  samples for optimization
- Regularized policy optimization can improve sample efficiency
- The method of regularization can induce a geometry on the loss surface that is often underexploited

## Contributions

- Introduce a local similarity measure between behavioral distributions based on the Wasserstein Information Matrix (WIM)
- Use a low rank approximation of the WIM to derive an efficient Wasserstein natural gradient (WNG) for RL
- Advantage over KL-based methods on problems with deterministic solutions.
- WNG + policy gradients → WNPG  
WNG + evolution strategies → WNES [1]
- Improved results on challenging continuous control tasks.

# RL Background



- Assume an agent is acting in an MDP  $(\mathcal{S}, \mathcal{A}, r, p, \gamma)$
- Running the policy in an episodic/finite horizon task of length  $T$  produces trajectories  $\tau = (s_1, a_1, r_1, \dots, s_T, a_T, r_T)$
- Maximize  $\mathbb{E}_\pi [Z(\tau)] = \mathbb{E}_\pi [\sum_t \gamma^t r_t]$

# Regularized Policy Optimization

- Consider policy gradients on some parametric policy  $\pi_\theta(a_t|s_t)$ :

$$F(\theta) = \mathbb{E}_\pi \left[ \sum_t \gamma^t r_t \right] \implies \nabla_\theta F = \mathbb{E}_\pi \left[ Z(\tau) \sum_t \nabla_\theta \log \pi_\theta(a_t|s_t) \right]$$

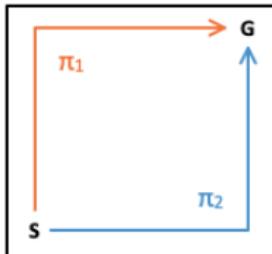
- Problem:** *expensive* to sample new trajectories, but re-using for multiple updates degrades optimization...
- Solution:** Only take large steps that change *behavior* the least:

$$\text{maximize}_\theta F(\theta) - \beta D(\pi_{\theta_k}(\cdot|s_t) || \pi_\theta(\cdot|s_t))$$

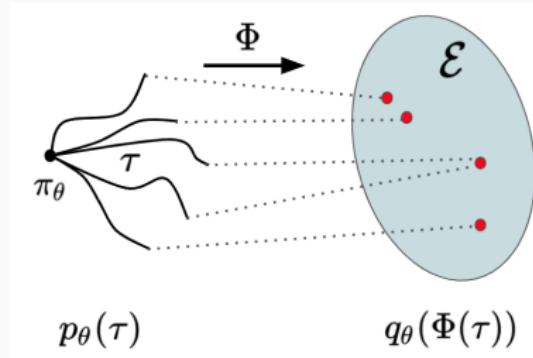
- ⇒ We can make multiple updates with the same trajectories

# Behavioral Geometry

- Local action distributions don't always reflect global *behavior*:



- How can we capture behavioral similarity? *Embed* trajectories [2]



- How to compare? Measure WD between embedding distributions

## Divergences Measures $\Rightarrow$ Natural Gradients

- Euclidean gradient: Steepest direction wrt the  $L_2$  distance

$$g^E = \underset{u}{\operatorname{argmax}} F(\theta) + \nabla F^\top u - \frac{1}{2} \|u\|^2 \approx \underset{u}{\operatorname{argmax}} F(\theta + u) - \frac{1}{2} \|u\|^2$$

- Fisher natural gradient: Steepest direction wrt the  $KL$

$$g^F = \underset{u}{\operatorname{argmax}} F(\theta) + \nabla F^\top u - \frac{1}{2} u^\top G_F u \approx \underset{u}{\operatorname{argmax}} F(\theta + u) - \frac{1}{2} KL[\pi_{\theta+u} \| \pi_\theta]$$

- Wasserstein natural gradient: Steepest direction wrt  $W_2$

$$g^W = \underset{u}{\operatorname{argmax}} F(\theta) + \nabla F^\top u - \frac{1}{2} u^\top G_W u \approx \underset{u}{\operatorname{argmax}} F(\theta + u) - \frac{1}{2} W_2(\pi_{\theta+u}, \pi_\theta)$$

$$\Rightarrow g^W = G_W^{-1} \nabla F$$

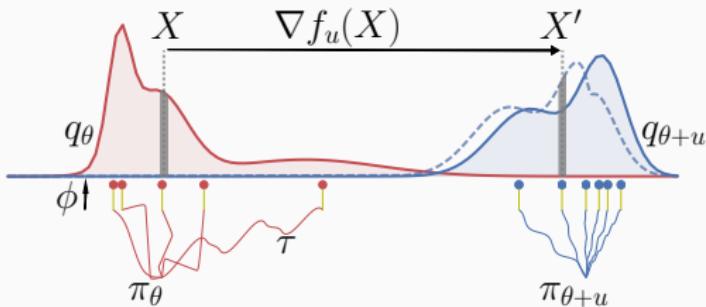
# Policy Optimization Using Behavioral Geometry

- The WD penalty captures a policy's global behavioral geometry [1]

$$\operatorname{argmax}_{\theta} F(\theta + u) + \frac{1}{2} \beta W_2^2(q_{\theta+u}, q_\theta) \quad (o.1)$$

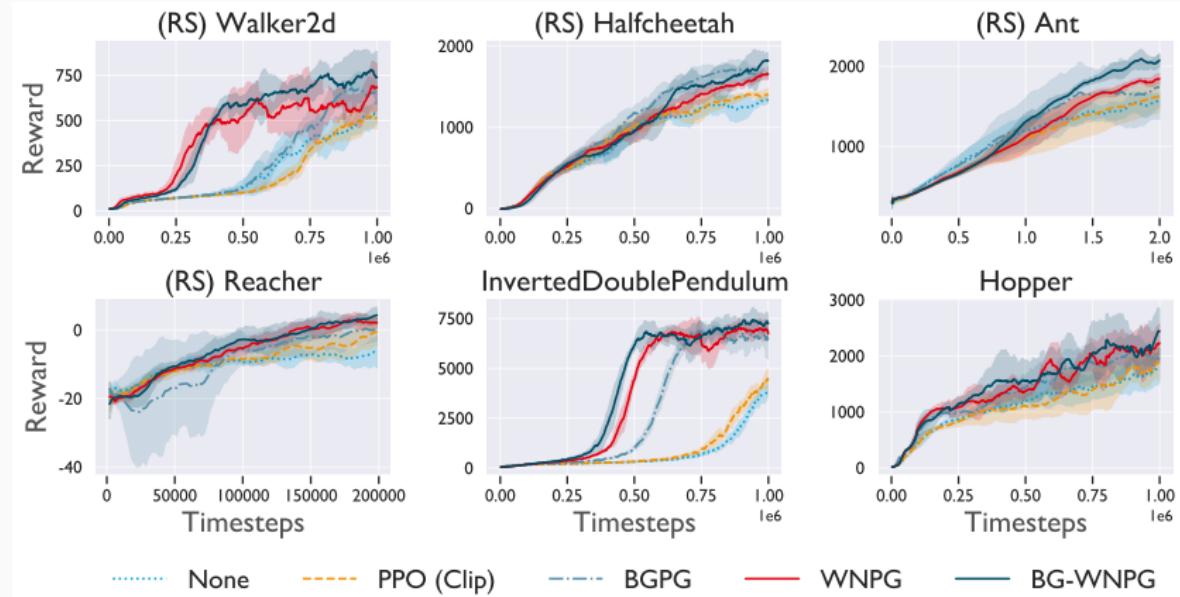
- The WNG captures the local behavior of a policy:

$$W_2^2(q_{\theta+u}, q_\theta) \simeq \mathbb{E}_{q_\theta} [\|\nabla f_u(X)\|^2] = u^\top G_W u \Rightarrow g^W = G_W^{-1} \nabla F \quad (o.2)$$



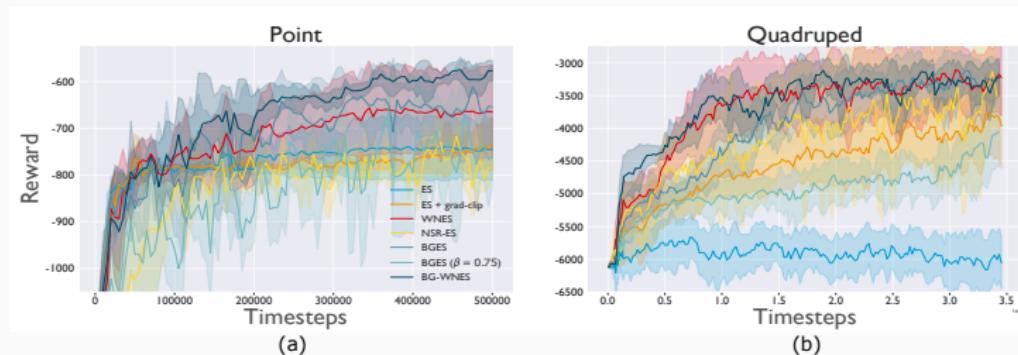
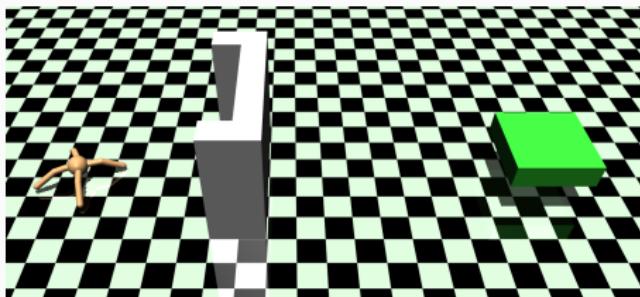
# Wasserstein Natural Policy Gradients

- Apply behavioral WNG to policy gradients (PG):  
(1)  $F(\theta)$  (**WNPG**)      (2)  $F(\theta) - \beta W_2(q_\theta, q_{\theta+u})$  (**BG-WNPG**)



# Wasserstein Natural Evolution Strategies

- Apply behavioral WNG to evolution strategies (ES):  
(1)  $F(\theta)$  (**WNES**)      (2)  $F(\theta) + \beta W_2(q_\theta, q_{\theta+u})$  (**BG-WNES**)



- [1] M. Arbel, A. Gretton, W. Li, and G. Montufar. Kernelized wasserstein natural gradient. In *International Conference on Learning Representations*, 2020.
- [2] A. Pacchiano, J. Parker-Holder, Y. Tang, A. Choromanska, K. Choromanski, and M. I. Jordan. Learning to score behaviors for guided policy optimization. *arXiv preprint arXiv:1906.04349*, 2019.